SAR TOMOGRAPHY OPTIMIZATION BY INTERIOR POINT METHODS VIA ATOMIC DECOMPOSITION - THE CONVEX OPTIMIZATION APPROACH

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ABSTRACT

In the Multi-Baseline SAR tomography remote sensing technique, the tomographic resolution is proportional to the vertical aperture component of the synthetic antenna. In order to avoid the problem of obtaining aliased tomographic results when designing multi-baseline SAR acquisition geometries using the fewest number of repeated radar tracks, it is necessary to process the data-set by advanced signal processing techniques that can properly process coherent and distributed composed environments SAR data. In this paper the Digital Gabor Transform (DGT) decomposition for sparsity seeking and the Compressed Sensing (CS) for signal recovery techniques performance will be analyzed. Recovery in highly over-complete dictionaries leads to large-scale optimization problems that can be successfully reached specially because of recent advances in linear and quadratic programming by Interior Point Methods (IPM). This paper considers the Convex Optimization (CVX) tomographic solution in order to process multi-baseline datasets over forested environments, in a Fourier under-sampled configuration. In this situation, the vertical reflectivity function is in a smooth domain. The DGT is a suitable method in order to generate an over-complete dictionary for sparsity seeking. The CVX Second Order Cone Programming Solution (SOCPs) by IPM using a generic log-barrier algorithm has been tested in order to optimize the dictionary atoms. In particular the following recovery technique has been implemented: l1 norm minimization with quadratic constraints (L1QC). This technique has been validated over real forested areas pointing out the better performance of the proposed solution in such a particular environment.

Index Terms — SAR, Tomography, Digital Gabor Transform (DGT), atomic decomposition, Convex Optimization (CVX), Second Order Cone Programming (SOCP), Interior Point Methods (IPM), Quadratic Constrained Quadratic Programming (QCQP), Path Following, Log Barrier, Newton Iteration, Conjugate Gradient (CG), Compressed Sensing (CS)

1. INTRODUCTION

SAR TOMOGRAPHY extends the synthetic aperture focusing principles in the elevation direction. In order to obtain a great vertical resolution so to distinguish among different scattering events inside a complex scattering scenario like forests, it is necessary to design vertical synthetic aperture antenna with a great extension. Indeed, to avoid aliasing problems. In order to achieve good tomographic results, it is necessary to acquire a large number of parallel tracks distributed within the vertical aperture interval. As a consequence the multi-pass SAR acquisitions, spanned during a long time frame, can induce the so called temporal decorrelation effect, that can defocus the tomographic map [1 2]. in order to avoid this, it is necessary to exploit repeated pass multi-baseline SAR acquisitions with a minimum number of tracks [3], and after processing the acquired data with powerful signal processing techniques so to adequately compensate aliasing and resolution loss [4 5] Traditional approaches of signal or image reconstruction from measured data follow the Shannon sampling theorem, which states that the sampling rate must be twice the highest frequency standing inside the signal. This suggests that the number of collected samples (observations) of a discrete finite-dimensional signal should be a great number in order to reconstruct the vertical profile in all its length without any aliasing effects. Compressed Sensing (CS) provides a new approach to perform a sparse data reconstruction, as required by multi-pass tomography in under-sampled configuration, which overcomes this common wisdom. The theory predicts that certain signals or images can be recovered from what was previously believed to be highly incomplete measurements. These kind of problems fall in two classes: those which can be recast as linear programming, and those which can be recast as a Second Order Cone Programming (SOCP) approach [6 – 16]. The intermediate step, between the recovery made by the Compressed Sensing and the existence of a smooth function that is therefore not sparse, is to find a base through which this function is expressible as a sum of sparse functions. This paper considers the DGT decomposition in order to research sparsity and each function of the dictionary has been recovered by IPM. In this section some tomographic results are reported in order to validate the IPM optimization algorithm on airborne tomographic data sets. The validation has been done on the TropiSAR campaign over the France Guiana acquired in the P band. The above described multi-baseline dataset has been processed in order to research the differences existing between the following two tomographic super-resolution methods: the Capon filter and the IPM methods. The target of this letter is to recast the above described Convex Optimization algorithm as a novel super-resolution method suitable to process environments prevalently constituted by forests. The SOCP problem treated in this paper is closely related to a quadratic programing class and stands to have the following standard form:

$$\begin{aligned} &\text{minimize} f_0(\boldsymbol{x}) \\ &\text{subject to} f_i(\boldsymbol{x}) \leqslant b_i; \quad i=1,\ldots,m \end{aligned} \tag{1}$$

Where $\boldsymbol{x}=(x_{1,}\ldots,x_n)$ is the optimization variable, $f_0:\mathbb{C}^n\rightarrow\mathbb{C}$ is the objective function and $f_i:\mathbb{C}^n\rightarrow\mathbb{C}$ the are the inequality constrained functions and the parameters $c_{1,}\ldots,c_m$ are the bounds of the constraints. Considering a quadratic programming approach, the following recovery method from noisy data was used:

$$\min\|\boldsymbol{x}\|_1 \text{subject to} \|\boldsymbol{y}-\boldsymbol{A}\boldsymbol{x}\|_2\leqslant\epsilon \tag{2}$$

where ε bounds the data noise amount.

This paper considers the DGT dictionary in order to research the sparsity condition. Each such dictionary D is a collection of waveforms $(\boldsymbol{\phi})_\gamma\in\Gamma$ where γ is the wave parameter. The decomposition of a signal vector **s** can so be:

$$\boldsymbol{s}=\sum_{\gamma\in\Gamma} a_\gamma \boldsymbol{\phi}_\gamma \tag{3}$$

The considered dictionaries are over-complete because they merges complete dictionaries, forming a mega-dictionary consisting of several types of waveforms. Such defined decomposition is non-unique because some elements in the dictionary have representations in terms of other elements. Because of the upon non-uniqueness characteristic, an adaptation possibility exists in order to research the following tomographic conditions:

- *Sparsity*: we should obtain the sparsest possible representation of the object, the one with the fewest significant coefficient;
- *Super-resolution: we should obtain a resolution of sparse objects in a much higher resolution if compared with the traditional non-adaptive approaches.*

The DGT Decomposition of signals over family of functions that are well localized both in time and frequency has found applications in SAR Tomography DSP. In this paper the DGT transforms are exampled as tomographic sparsity seekers and the results have been studied thoroughly. To extract information from complex signals, it is often necessary to adapt the time-frequency decomposition to the particular signal structures. A general time-frequency atoms family can be generated by scaling, translating and modulating a function $g(t)\in L_2(\mathbb{C})$, where g(t) is complex and continuously differentiable, where $\|g(t)\|=1$ and $g(0)\neq 0$ The DGT of an 1-D signal vector is defined as follows: Consider the upon considered window function **g** and the input signal ***f*** of length *L* and define the parameter N=L/a. The DGT of the 1-D signal vector ***f*** is given by:

$$\boldsymbol{c}(m+1,n+1)=\sum_{i=0}^{L-1}\boldsymbol{f}(l+1)\overline{\boldsymbol{g}(l-an+1)}e^{\frac{-2\pi\, ilm}{M}} \tag{4}$$

In (4) ***f*** is the input data vector, ***g*** is the window function, *a* is the shift length, M is the number of channels, L is the length of the transform to perform and c is an P×N array of coefficients. The output is a vector or a matrix in a rectangular layout. The length of the transform is the smallest multiple of a and M that is larger than the signal. The function ***f*** is zero-extended to the length of the transform. The window vector ***g*** is a vector of numerical values, in this paper, a Gaussian window has been used. In order to reconstruct the recovered signal, the following Inverse Digital Gabor Transform (IDGT) has to be performed:

$$\boldsymbol{f}(l+1)=\sum_{n=0}^{N-1}\sum_{m=0}^{M-1}\boldsymbol{c}(m+1,n-1)e^{\frac{2\pi\, ilm}{M}}\cdot\boldsymbol{g}(l-an+1). \tag{5}$$

In (5) ***c*** is the recovered array of coefficient, **g** is the same window function used to transform the original signal and *a* is the time shift length.

2. EXPERIMENTAL RESULTS

In this section some tomographic results are reported in order to validate the IPM optimization algorithm on airborne tomographic data sets. The validation has been done on the TropiSAR campaign over the France Guiana acquired in the P and L-bands. The above described multi-baseline dataset has been processed in order to research the differences existing between the following two tomographic super-resolution methods: the Capon filter and the IPM optimization method. The target of this letter is to recast the above described SOCP algorithm as an efficient super-resolution method suitable to process environments prevalently constituted by forests.

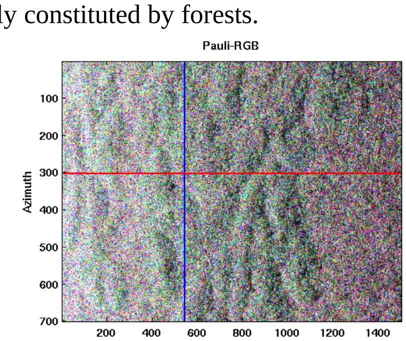


Fig.1: In this picture a full-polarimetric vegetation environment Range-Azimuth SAR image is depicted. The

blue and the red lines are the two trajectory where the tomographic processing has been performed.

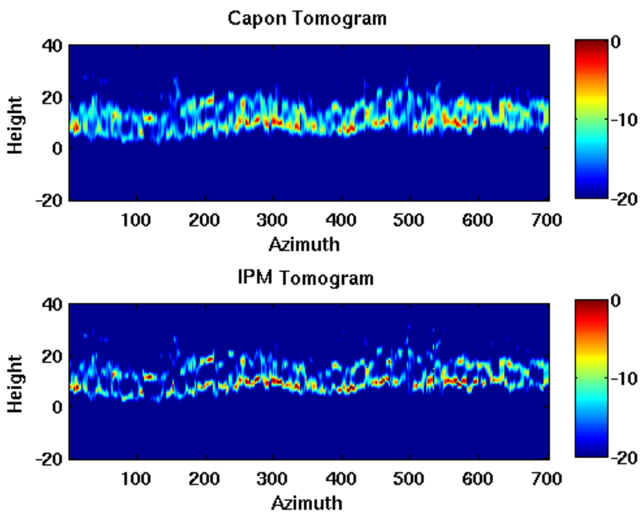


Fig.2: In this figure the Capon and the SOCP log-barrier optimization tomographic results are compared. The tested environment is referred to the one tracked in the azimuth direction by the blue line in Fig. 1 depicted. The IPM result is better because no noise and side-lobes are present. The vegetation consistency and a better ground top-height separation are observable.

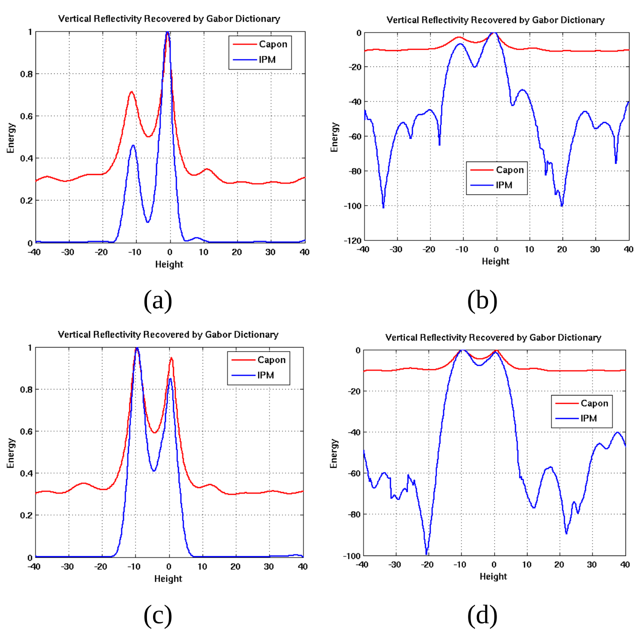


Fig.3: Vertical cross-range profiles (in meters) referred to the tomographic results in Fig. 2 (UP) and (DOWN) depicted. All plots has been normalized and rescaled to the dB energy scale and are referred to the vertical lines linked in Fig.2 by the red color as (a) and (b). The red function is referred to the Capon filter result and the blue function is the result optimized by the IPM method.

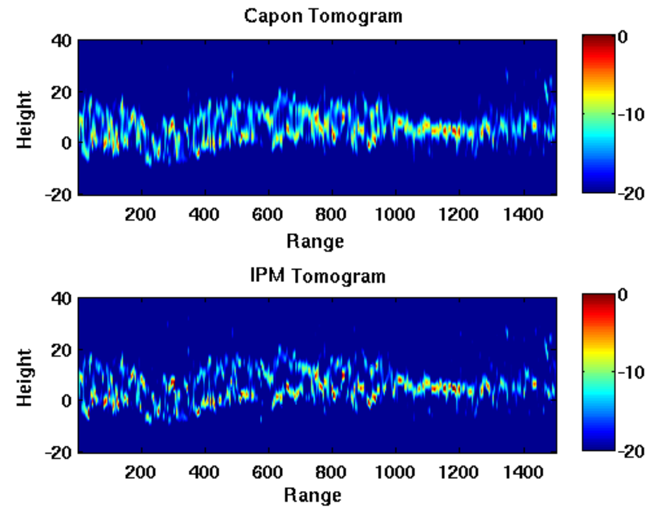
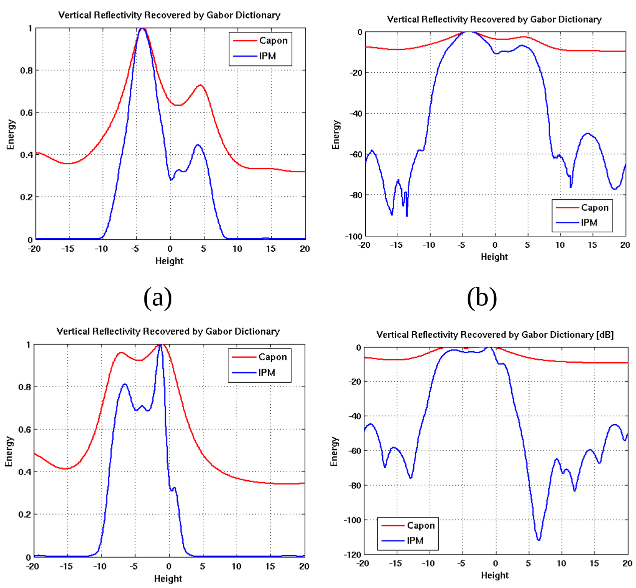


Fig.4: In this figure the Capon and the SOCP log-barrier optimization tomographic results are compared. The tested environment is referred to the one tracked in the range direction by the red line in Fig. 1 depicted. The IPM result is better because no massive noise and side-lobes are present. The vegetation consistency and a better ground top-height separation are observable.

The experimental results consists in two tomographic processing, the first one consists in a range-height plane, in Fig. 2 visible and the second consists in an azimuth-height plane in Fig. 4 visible. In Fig. 1 the polarimetric range-azimuth energy map is depicted, the blue and red lines drives the paths from witch the tomographic planes has been estimated. In Fig. 3 (a), (b), (c) and (d) and Fig. 4 (a), (b), (c), and (d) four vertical reflectivity profiles are depicted, the functions are complex vectors and the modulus in the linear and logarithmic scale has been presented. The functions are referred to the red vertical lines in Fig. 2 and Fig. 3 depicted. The IPM results are estimated after performing a DGT over-complete decomposition. The IPM result is better than the Capon result because the S/N ratio is massive higher and it is observable a vertical resolution enhancement

3. CONCLUSIONS

In this paper a novel super-resolution method to perform SAR tomography applied to multi-baseline airborne data was proposed. The method is the Second Order Cone Programming Interior Point Method optimization that has been validate on the TropiSAR data set collected in the P-band by ONERA over Paracou situated in the French Guyana. This paper shows several contrasted results obtained by two super-resolution algorithms. The first one is the Capon non parametric spectral estimator, and the second is the Convex Optimization algorithm (CVX) that uses second order cones as constraints functions that implements the Gabor over-complete Dictionary. The validation was performed processing the above mentioned multi-baseline dataset. The two signal processing techniques has been compared in order to validate the IPM signal processing ideal to optimize tomographic profiles in a smooth configuration. Observing the results the considered signal processing method confirms the above mentioned qualities.



(a) (b)

(c) (d)

Fig.5: Vertical cross-range profiles (in meters) referred to the tomographic results in Fig. 2 (UP) and (DOWN) depicted. All plots has been normalized and rescaled to the dB energy scale and are referred to the vertical lines linked in Fig.2 by the red color as (a) and (b). The red function is referred to the Capon filter result and function is the result optimized by the IPM method, after a DGT over-complete Decomposition. The IPM result is better than the Capon result because the S/N ratio is massive higher and it is observable a vertical resolution enhancement.

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